

## **Extended-Range Prediction with Low-Dimensional, Stochastic-Dynamic Models: A Data-driven Approach**

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### **LONG-TERM GOALS**

The long-term goal of this project is to quantify the extent to which reduced-order models can be used for the description, understanding and prediction of atmospheric, oceanic and sea ice variability on time scales of 1–12 months and beyond.

### **OBJECTIVES**

Demonstrate the ability of linear and nonlinear, stochastic-dynamic models to capture the dominant and most predictable portion of the climate system's variability. Improve the understanding and prediction of the low-frequency modes (LFMs) of variability such as the Madden-Julian Oscillation (MJO), El Niño–Southern Oscillation (ENSO), North Atlantic Oscillation (NAO) and Pacific–North American

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(PNA) pattern. Validate low-dimensional models (LDMs) based on data sets from observations, reanalyses and high-end simulations.

## APPROACH

The major theoretical problem we propose to address is the fact that in a nonlinear system there is a continuum of scales, from the fastest and shortest to the slowest and longest: typically, there is no gap or separation between these scales. Recent ideas and methods from statistical physics and dynamical systems theory — associated with the names of H. Mori and R. Zwanzig — allow one to treat the effect of the larger scales on the smaller ones as memory effects. We propose numerically efficient methods for treating these effects, thus further improving a panoply of methods for model reduction and the application of the reduced models thus obtained to prediction.

The technical approaches include the use of linear and nonlinear, stochastic-dynamic models that capture the dominant and hopefully most predictable portion of the climate system’s variability. In particular, the low-order models we propose rely heavily on the proper identification and robust description of LFM’s. The expertise of the two teams participating in the project, at UCLA and at Columbia University (CU), includes pioneering work on the use of LFM’s in extended-range prediction, as well as extensive experience with the operational performance of a variety of predictive models and methods on several time scales.

Our approach is data-driven and does not require knowledge of the governing equations. Data — whether observations from nature or GCM output — are used to estimate both the model’s low-order, deterministic part and its driving noise. These LDM models can be linear (Penland, 1989) or nonlinear, as in the recently developed *empirical model reduction* (EMR) methodology (Kondrashov et al., 2006, 2011, 2005; Kravtsov et al., 2005, 2009). EMR models account for nonlinearity, serial correlation in the noise residue and memory effects, and was shown to be particularly effective when there is no *spectral gap* (time scale separation) between slow and fast variables.

A key advantage of the data-driven approach is that there are no explicit assumptions on the full dynamical equations, and, in fact, the equations do not have to be known. The main difficulty in this approach, though, is that the more realistic an LDM, the more parameters have to be estimated from the limited data; still, when applying well-established regularization techniques, robust parameter estimates can be obtained.

Relatively simple data-driven linear LDMs had already success simulating and predicting a wide range of climate phenomena including ENSO, tropical Atlantic sea surface temperatures (SSTs), storm track variability and the Madden-Julian Oscillation (MJO) (Jiang et al., 2008; Penland and Matrosova, 1998; Winkler et al., 2001). Barnston et al. (2012) analyzed the real-time forecast performance of EMR-ENSO model (Kondrashov et al., 2005) in IRI’s ENSO multi-model prediction plume for the 2002–2011 interval, and have found that it “has the highest seasonally combined correlation skill among the statistical models, exceeded by only a few dynamical models”.

By extending the EMR results of Kondrashov et al. (2005) beyond the sampled-PDF approach, Chekroun et al. (2011) have developed a pathwise *past noise forecasting* (PNF) method of prediction for nonlinear stochastic systems that exhibit LFM’s. The PNF method exploits information on the estimated noise path on which the inverse stochastic model lives, and the LFM’s resulting from the interaction between the model’s deterministic, nonlinear components and the stochastic ones. The PNF method can

provide retrospective ENSO forecasts at 14–16 months with an accuracy that is comparable to the one obtained at 6–8 months by the methods — dynamical as well as statistical — that are currently available in the IRI plume. We plan to extend this PNF method to other applications and LDMs.

*Theoretical track.* Provide theoretical guidance for practical development of next-generation LDMs for climate modeling and prediction by developing further and applying the Mori–Zwanzig (MZ) formalism. In the MZ approach, one considers the full nonlinear dynamics as a sum of three terms: (i) the projected dynamics that governs the resolved variables, (ii) the orthogonal dynamics of the unresolved ones, and (iii) their interactions through memory terms that arise in an integral form; the latter are given by repeated convolutions between decaying memory kernels and the resolved modes (Chorin et al., 2002, 2006; Chorin and Stinis, 2006; Grabert, 1982; Stinis, 2006). These integral terms appear in the projected dynamics in order to represent the interaction terms between the resolved and unresolved scales, without assuming a gap or separation between these scales.

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*Methodological improvements.* Improve further the EMR and PNF methodology to develop computationally efficient and robust LDMs. We plan to develop the new-generation EMR models in which the generalized energy is conserved. This development will follow the spectral expansion of the governing equations in most fluid-dynamical problems, which imposes additional properties on the model coefficients (Ghil and Childress, 1987). We are in the process of introducing appropriate linear constraints in the least-square estimation of the EMR coefficients, thus reducing effectively their number and achieving therewith a form of regularization. The choice of basis in which the resolved variables are represented will also be investigated accordingly, since the typical choice of the EOFs as basis functions for the resolved dynamics is not supported by theoretical guidance.

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*ENSO and coupled variability on seasonal time-scales.* Improve understanding and prediction of ENSO, tropical Atlantic variability (TAV), and the Indo-Pacific coupled mode (IPT).

Mark A. Cane, Dake Chen, Alexey Kaplan.

*Sea ice.* Predict Arctic sea ice on intraseasonal to seasonal time scales, improve Antarctic sea ice forecasts and assess the impact of sea ice on mid-latitude predictability.

Xiaojun Yuan, Dake Chen.

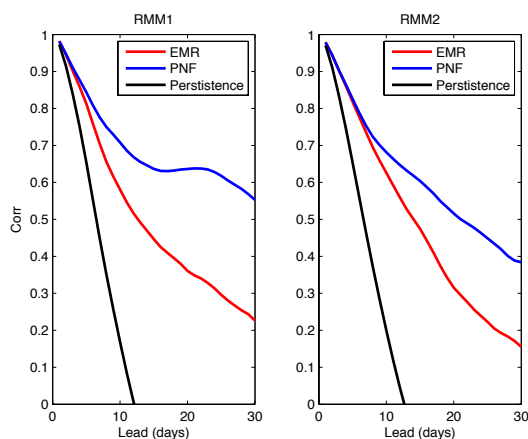
*MJO.* Incorporate slow moisture dynamics into empirical prediction models. Use observations to improve low-order dynamical MJO models.

Adam Sobel, Daehyun Kim.

*Extratropical variability and predictability.* Determine the extent to which extratropical monthly and seasonal LFV — i.e., PNA, NAO, as well as other regional blocking patterns — can be skillfully predicted from one to 12 months in advance in a LDM including ENSO, MJO and stratospheric linkages. Provide a statistical characterization of extratropical storms and extremes and link these to LFV modes.

Mingfang Ting, Yochanan Kushnir, Andrew W. Robertson.

*Downscaling weather and seasonal climate.* Use LDMs to relate near-surface temperature and precipitation with predictable climate forcing such as the SST field. Explore the development of statistical tropical cyclone forecasts on the intraseasonal time scale, based on LDM forecasts of ENSO



**Figure 1:** Prediction skill for the (RMM1, RMM2) pair of EOF-filtered indices<sup>3</sup> of the Madden-Julian Oscillation (MJO): EMR (red) and PNF (blue) prediction, while the black curve shows damped persistence for a base comparison.

and the MJO. Target tropical monsoon climates, with particular attention to environmentally and societally relevant sub-seasonal characteristics of weather. The characteristics of interest include the start and end dates of the monsoon seasons, as well as extremes of rainfall and drought within the monsoon season.

Andrew W. Robertson, Suzana J. Camargo, Yochanan Kushnir, Michael K. Tippett.

## WORK COMPLETED

**Year 1 plans:** Complete preliminary study of EMR-PNF prediction for the MJO, and initiate development of energy-conserving EMR framework at UCLA. Interact closely with the corresponding MJO and sea ice sub-teams at Columbia University (CU) to initiate application of these methods to sea ice variability and prediction.

Kick-off meeting planned for October 5 in Palisades, NY, to bring together UCLA and CU researchers with ONR program managers.

Post-doctoral research positions at CU have been advertised, and the post-doctoral position at UCLA will be advertised as soon as financial arrangements have been completed.

## RESULTS

- The constructed analogue (CA) forecast method gives weights to past states according to the extent to which they represent analogues of a future state (van den Dool, 1994, 2006). CA is used in operational seasonal forecasting and had been described as having properties that are distinct from those of linear regression. We have shown that standard implementations of the CA method correspond to principal component regression and ridge regression (Tippett and DelSole, 2012).
- A quadratic, three-level EMR model was developed to model and predict the leading pair (RMM1, RMM2) of real-time, daily MJO indices. The EMR model was fitted, and the PNF method was trained, on the first 30 yr of the Real-time Multivariate MJO Index (RMM) record; the respective forecasts were validated on the record's last 3.5 yr. The cross-validated prediction skill of the PNF method is notably better than the EMR alone (blue vs. red curve in Fig. 1) and compares favorably with both the statistical and dynamical results of Kang and Kim (2010).

<sup>3</sup><http://cawcr.gov.au/staff/mwheeler/maproom/RMM/>

- As the Madden-Julian oscillation (MJO) moves eastward from the Indian to the Pacific ocean, it typically accelerates, becomes less strongly coupled to convection, and becomes more similar to a dry Kelvin wave. This transition was analyzed using observations of outgoing longwave radiation and ERA Interim reanalyses of surface pressure and 850 hPa zonal wind. Transitions are well defined, with distinct disturbances on either side of the transition whose identities as MJO or Kelvin waves are clear. Transitions occur over a wide range of longitudes in different events, all the way from the Eastern Indian to the Central Pacific oceans.

We had shown in a simple linear model of intraseasonal moisture modes (Sobel and Maloney, 2012) that wind-evaporation feedbacks induce westward propagation in an eastward mean low-level flow. We consider here additional processes that provide effective sources of moist static energy to the disturbances, and which also depend on the low-level wind. Several processes can act as positive sources in perturbation easterlies: zonal advection (if the mean zonal moisture gradient is eastward), modulation of synoptic eddy drying by the MJO-scale wind perturbations, and frictional convergence. If the sum of these is stronger than the wind-evaporation feedback — as observations suggest may be the case, though with considerable uncertainty — the model produces unstable modes that propagate weakly eastward, relative to the mean flow.

## IMPACT/APPLICATIONS

## RELATED PROJECTS

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